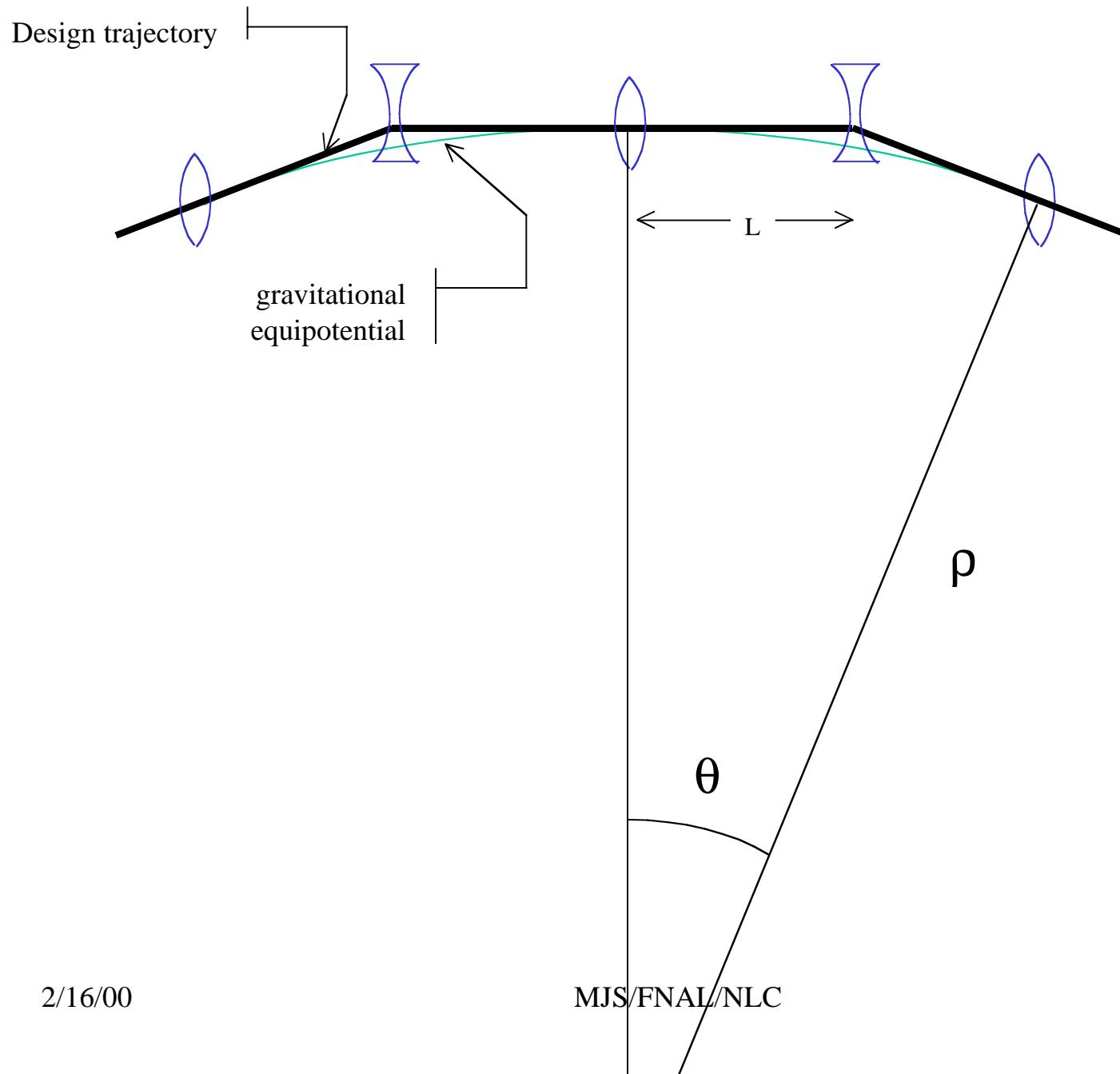
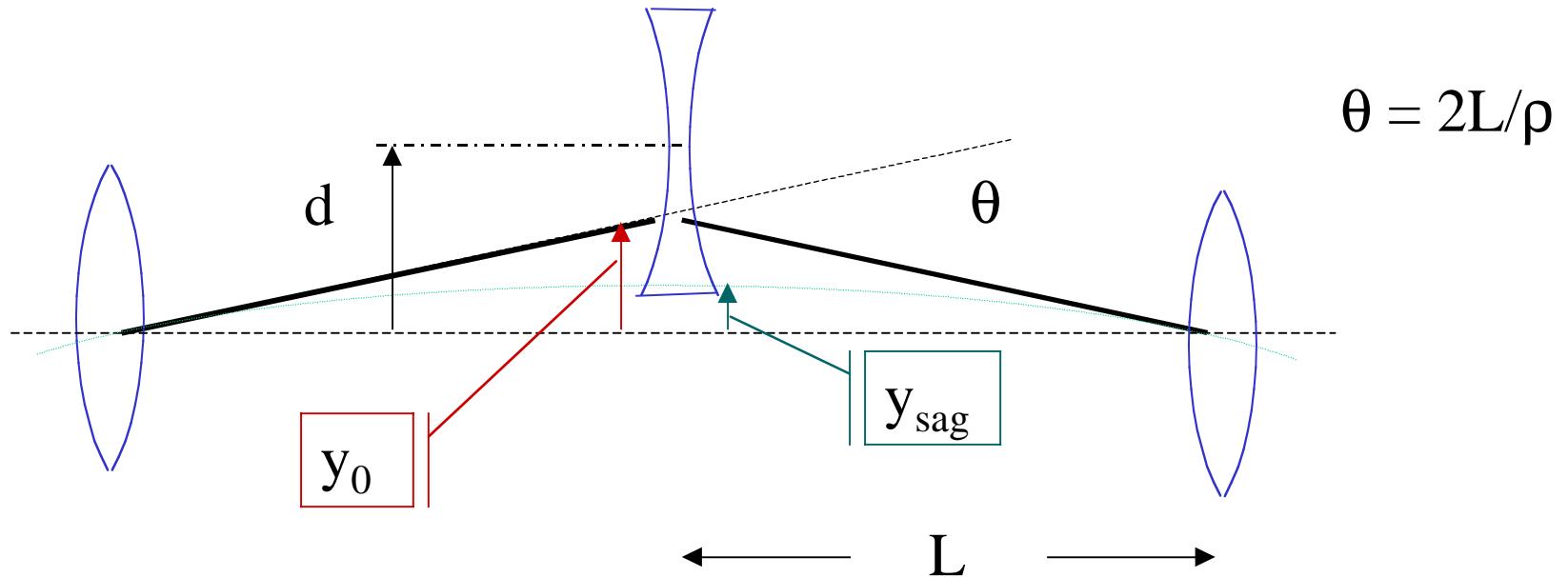


Laser Straight vs. Curved “Linac”

- Follow gravitational equipotential
- Use displaced quadrupoles to generate bend
- To get order of magnitude:
 - Assume periodic structure
 - Steer with “D” quads, through center of “F” quads (minimizes dispersion)
 - Compute periodic dispersion function, assuming can match into this somehow upstream





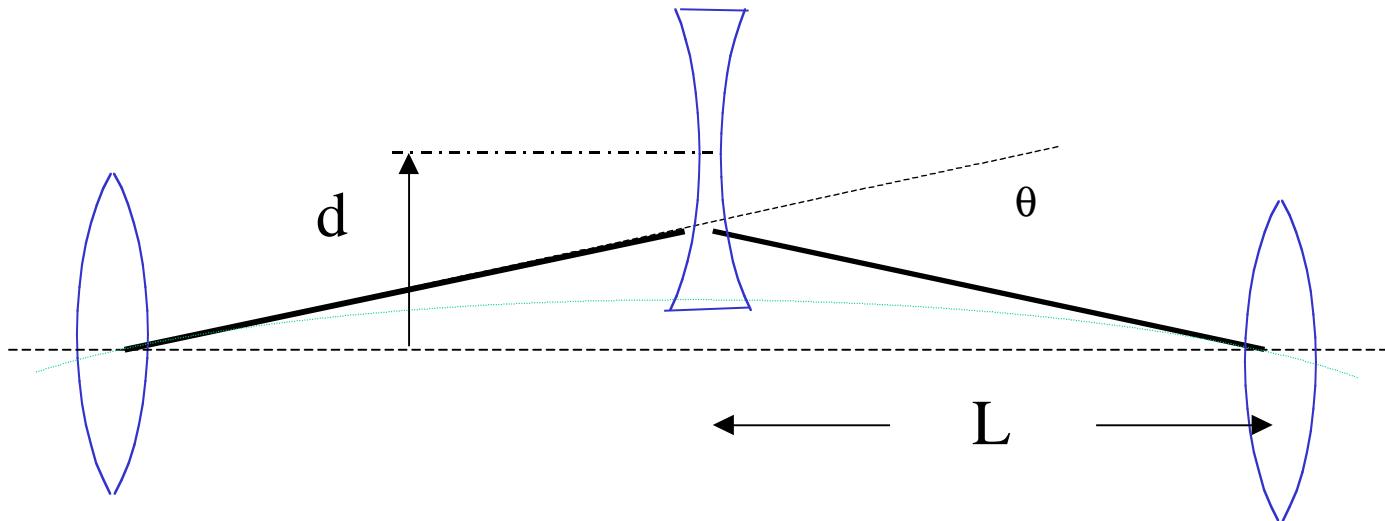
$$y_0 = L \cdot \frac{\theta}{2} \approx L^2 / \rho$$

$$\frac{d - y_0}{F} = \theta \Rightarrow d = \theta F (1 + L/2F)$$

$$y_{\text{sag}} = \rho \cdot \left(1 - \cos \frac{\theta}{2}\right) \approx \frac{1}{8} \rho \theta^2 = \frac{L^2}{2\rho}$$

$$\Rightarrow d = \frac{L^2}{\rho} \left(1 + \csc \frac{\mu}{2}\right)$$

$$\Rightarrow d - y_{\text{sag}} = \frac{L^2}{2\rho} \left(1 + 2 \csc \frac{\mu}{2}\right)$$



$$\begin{pmatrix} 1 & 0 \\ -\frac{q}{2} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{q}{2} & 1 \end{pmatrix} \left[\begin{pmatrix} \check{D} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta/2 \end{pmatrix} \right] = \begin{pmatrix} \hat{D} \\ 0 \end{pmatrix}$$

$$\hat{D} = \frac{F\theta}{\sin \frac{\mu}{2}}$$

or,

$$\check{D} = \frac{F\theta}{\sin \frac{\mu}{2}} \left(1 - \sin \frac{\mu}{2} \right)$$

$$\hat{D} = \frac{L^2}{\rho \sin^2 \frac{\mu}{2}}$$

$$\check{D} = \frac{L^2}{\rho \sin^2 \frac{\mu}{2}} \left(1 - \sin \frac{\mu}{2} \right)$$

Put in some numbers -- high energy end of linac:

$$L = 3*3*2 \text{ m} = 18 \text{ m}$$

$$\mu = 90^\circ$$

$$\rho = 6400 \text{ km}$$

... then,

$$\theta = 5.6 \text{ } \mu\text{rad}$$

$$y_0 = 51 \text{ } \mu\text{m}$$

$$y_{\text{sag}} = 25 \text{ } \mu\text{m}$$

$$d = 122 \text{ } \mu\text{m}$$

$$d - y_{\text{sag}} = 97 \text{ } \mu\text{m}$$

and

$$D_{\min} = 0.03 \text{ mm}, \quad D_{\max} = 0.10 \text{ mm}$$

For $dE/E = 2\%$ (BNS damping), and $\gamma\varepsilon_y = 0.1 \text{ } \mu\text{m}$, 100 GeV:

$$D_{\max}(dE/E) = 2 \text{ } \mu\text{m}, \quad \text{compared to} \quad (\varepsilon_y/\beta_{\max})^{1/2} = 5.6 \text{ } \mu\text{m}$$

Approximate radiated energy per quadrupole:

$$\Delta U = \frac{1}{2\pi} C_\gamma \left[\frac{(d - y_o)}{F \cdot L_{quad}} \right]^2 E^4 L_{quad}$$

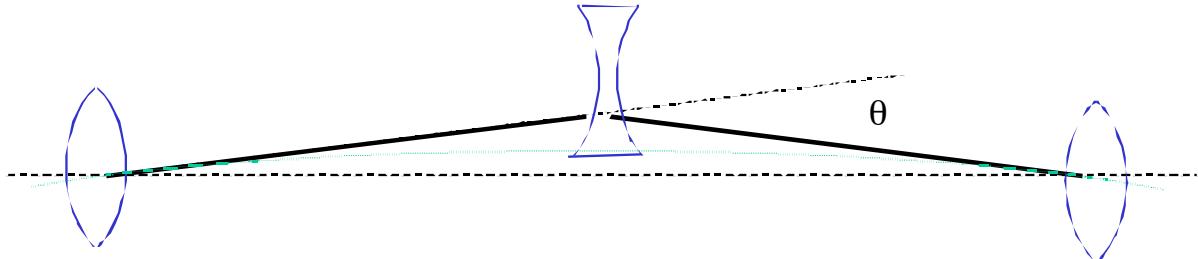
At 100 GeV, and assuming $L_{quad} = 0.8$ m,

$$\Delta U = 0.056 \text{ keV}$$

$$\Delta U/E = 5.6 \cdot 10^{-10}$$

Following a Geodesic

The concept... focusing quads are placed along a constant gravitational potential -- "level" -- and defocusing quads are displaced in order to provide the bending of the beam toward the next focusing quad.



$$\text{Distance between quads: } L := 3 \cdot 3 \cdot (2 \cdot m) \quad \mu\text{m} := 10^{-6} \cdot \text{m}$$

$$\text{Cell phase advance: } \mu := 90 \cdot \text{deg}$$

$$\text{Radius of Earth: } \rho_e := 6400 \cdot \text{km} \quad \theta := \frac{2 \cdot L}{\rho_e} \quad \theta = 5.625 \cdot 10^{-6}$$

$$F := \frac{L}{2 \cdot \sin\left(\frac{\mu}{2}\right)} \quad F = 12.728 \cdot \text{m}$$

$$\text{Beam trajectory displacement at defocusing quad relative to line joining focusing quadrupoles: } y_0 := L \cdot \frac{\theta}{2} \quad y_0 = 50.625 \cdot \mu\text{m}$$

$$\text{Defocusing quadrupole displacement from line joining centers of focusing quadrupoles: } d := \theta \cdot F \cdot \left(1 + \frac{L}{2 \cdot F}\right)$$

$$d = 122.22 \cdot \mu\text{m}$$

$$\text{Sagitta of geodesic curve joining centers of defocusing quadrupoles, and passing through angle theta: } y_{\text{sag}} := \frac{L \cdot \theta}{4} \quad y_{\text{sag}} = 25.313 \cdot \mu\text{m}$$

$$\text{Quadrupole offset relative to geodesic curve: } d - y_{\text{sag}} = 96.907 \cdot \mu\text{m}$$

Periodic dispersion generated by the offset quadrupole:
(assumes constant phase advance per cell...)

$$D_{\max} := \frac{L^2}{\rho_e \cdot \sin\left(\frac{\mu}{2}\right)^2} \quad D_{\min} := D_{\max} \cdot \left(1 - \sin\left(\frac{\mu}{2}\right)\right)$$

$$D_{\min} = 0.03 \cdot \text{mm}$$

$$D_{\max} = 0.101 \cdot \text{mm}$$

$$\beta_{\max} := 2 \cdot F \cdot \sqrt{\frac{1 + \sin\left(\frac{\mu}{2}\right)}{1 - \sin\left(\frac{\mu}{2}\right)}} \quad \beta_{\min} := 2 \cdot F \cdot \sqrt{\frac{1 - \sin\left(\frac{\mu}{2}\right)}{1 + \sin\left(\frac{\mu}{2}\right)}} \quad \beta_{\max} = 61.456 \cdot \text{m}$$

$$\beta_{\min} = 10.544 \cdot \text{m}$$

$$\alpha := \frac{\beta_{\max}}{2 \cdot F} \quad \alpha = 2.414$$

$$\text{keV} := 10^3 \cdot (1.6 \cdot 10^{-19}) \cdot \text{joule}$$

$$\text{MeV} := 10^3 \cdot \text{keV}$$

$$\text{GeV} := 10^3 \cdot \text{MeV}$$

$$\text{Beam energy: } E := 100 \cdot \text{GeV} \quad E_0 := 511 \cdot \text{keV} \quad \gamma := \frac{E}{E_0} \quad \gamma = 1.957 \cdot 10^5$$

$$\text{Energy Spread, from BNS damping: } \delta := 2\% \quad \text{Emittance: } \gamma \epsilon_y := 0.1 \cdot \mu\text{m}$$

$$\text{Beam Size from dispersion: } \delta \cdot D_{\max} = 2.025 \cdot \mu\text{m}$$

$$\text{Beam Size from betatron osc's: } \sqrt{\frac{\gamma \epsilon_y \cdot \beta_{\max}}{\gamma}} = 5.604 \cdot \mu\text{m}$$

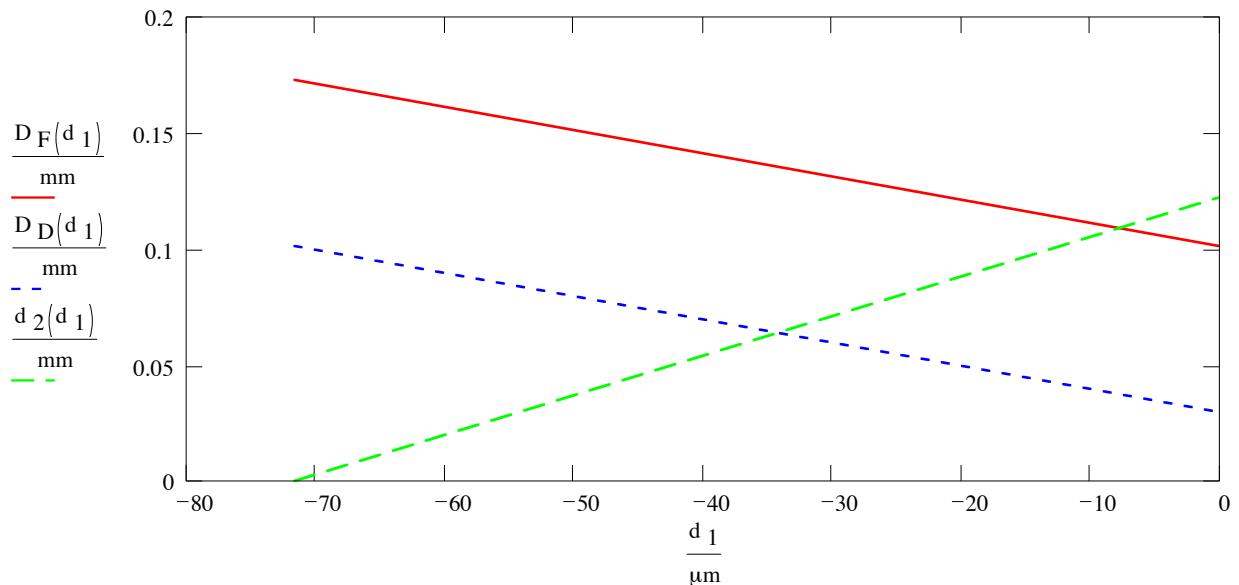
Suppose both D and F quads are displaced...

$$d_1 := -\theta \cdot F, -\theta \cdot F \cdot 0.001 \dots 0$$

$$D_F(d_1) := \frac{\theta \cdot F}{\sin\left(\frac{\mu}{2}\right)} - d_1$$

$$d_2(d_1) := (\theta \cdot F + d_1) \left(1 + \sin\left(\frac{\mu}{2}\right)\right)$$

$$D_D(d_1) := \frac{\theta \cdot F}{\sin\left(\frac{\mu}{2}\right)} \cdot \left(1 - \sin\left(\frac{\mu}{2}\right)\right) - d_1$$



$$r_0 := 2.81794092 \cdot 10^{-15} \cdot m \quad C_\gamma := \frac{4 \cdot \pi}{3} \cdot \frac{r_0}{E_0^3} \quad C_\gamma = 8.846 \cdot 10^{-5} \cdot \frac{m}{\text{GeV}^3}$$

$$L_{\text{quad}} := 0.8 \cdot m \quad \rho := \frac{F \cdot L_{\text{quad}}}{(d - y_0)} \quad \rho = 1.422 \cdot 10^5 \cdot m$$

Approx radiated energy per quad:

$$\Delta U := \frac{1}{2 \cdot \pi} \cdot C_\gamma \cdot \frac{1}{\rho^2} \cdot E^4 \cdot L_{\text{quad}}$$

$$\Delta U = 0.056 \cdot \text{keV}$$

$$\frac{\Delta U}{E} = 5.568 \cdot 10^{-10}$$

$$c := 2.99792458 \cdot 10^8 \cdot \frac{m}{sec}$$

$$\text{hbar} := 6.582122 \cdot 10^{-22} \cdot \text{MeV} \cdot \text{sec}$$

$$w_c := \frac{3}{2} \cdot \gamma^3 \cdot \text{hbar} \cdot \frac{c}{\rho}$$

$$w_c = 15.597 \cdot \text{keV}$$

Critical photon energy

$$w_{rms} := \sqrt{\frac{11}{27} \cdot w_c^2}$$

$$w_{rms} = 9.955 \cdot \text{keV}$$

rms photon energy

$$w_{ave} := \frac{8}{15 \cdot \sqrt{3}} \cdot w_c$$

$$w_{ave} = 4.803 \cdot \text{keV}$$

average photon energy

$$N := \frac{\Delta U}{w_{ave}}$$

$$N = 0.012$$

average no. photons emitted

Approximate value for script-H at middle of a "D" quad:

$$\text{script-H}_D := -\frac{D \frac{min^2}{min}}{\beta}$$

$$\Delta \sigma_y := \sqrt{\frac{1}{2} \cdot N \cdot \text{script-H}_D \cdot \beta \frac{min}{min} \cdot \frac{w_{rms}^2}{E^2}}$$

$$\Delta \sigma_y = 2.248 \cdot 10^{-7} \cdot \mu\text{m}$$

approx. emittanace increase per quad:

$$\Delta \epsilon_N := -\frac{\gamma \cdot \Delta \sigma_y^2}{\beta \frac{min}{min}} \quad \Delta \epsilon_N = 0 \cdot \mu\text{m}$$

$$E = 100 \cdot \text{GeV}$$